9/5/2016 الاشـــيم د. عرفــة محاضرة [١٠]

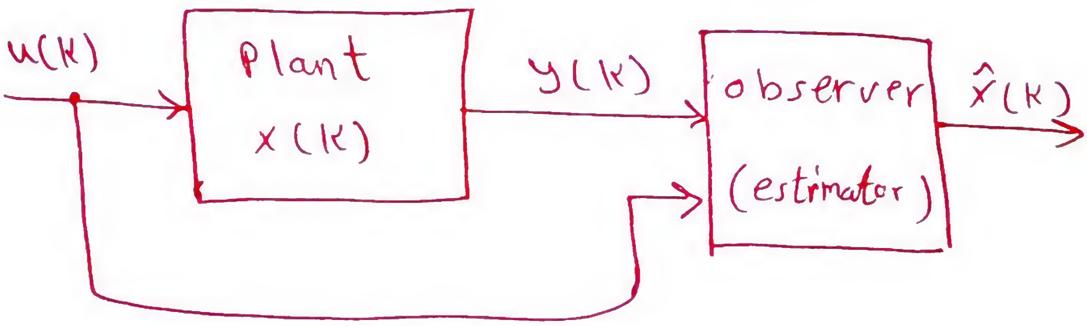
Observer Design:-

For state feedback control (Pole Placement design):

$$u(k) = -K\,x(k)$$

It's required for all states to be accessible for measuring.

In the case of some or all of the states are not accessible for measuring; an observer should be designed to estimate these states values. The estmation is done by the history of data for i/p and o/p



The observer equation:-

$$\hat{x}(k+1) = (A-\boxed{G}C)\,\hat{x}(k) + Bu(k) + Gy(k)$$

Gain Matrix

gain Matrix $(G) \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}$

order of the system

The gain matrix G is determined through the specs of the observer

as: - observer speed response
- transient
- settling time

$\Big\}\ \xi, \omega_n \Rightarrow$ desired poles of the observer

The Gain matrix G is determined by:

① desired ch. equation for the observer ($\alpha_o(z)$)

$$\alpha_o(z) = (z - P_1)(z - P_2) \cdots (z - P_n) = 0 \quad (1)$$

where $P_1, P_2, \ldots, P_n$ are the desired poles.

$$\hat{x}(k+1) = \underbrace{(A - GC)}_{\text{the observer ch. equation}} \hat{x}(k) + B u(k) + G y(k)$$

the observer ch- equation

$$|zI - A + GC| = 0 \quad (2)$$

by comparing the parameters of eqn (1) & (2)

we can get G

② using Ackerman's method:-

$$G = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

for $n = 2$ $\Rightarrow G = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

where M_o = observability matrix

$$\alpha_o(A) = \alpha_o(z) \Big|_{z=A}$$

$\alpha_o(z)$ = desired ch. eqn for observer

Ex: $x(k+1) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} u(k)$

$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$

① determine the gain matrix K such that the desired closed Loop poles are located at

$$z_{1,2} = 0.258 \pm j0.295$$

← from previous Lecture (9)

$K = (0.0775 \quad 0.3945)$

$K = (0 \quad 1)\, M_c^{-1}\, \alpha_c(A)$

$\alpha_c(z) = z^2 - 1.056z + 0.366$

$\alpha_c(A) = \begin{pmatrix} 0.31 & 1.888 \\ 0 & 0.31 \end{pmatrix}$

$M_c^{-1} = \begin{pmatrix} -0.25 & 0.75 \\ 0.25 & -0.25 \end{pmatrix}$

② design a full order observer such that $\xi = 1$ & the time constant of the observer is one half the time constant of the desired poles in ①

$T = 1$ sec.

$\boxed{\tau_o = \tfrac{1}{2}\tau_c}$

① ?

$r = e^{-\xi\omega_n T} = e^{-T/\tau}$

$\theta = \pm\,\omega_n T\sqrt{1-\xi^2}$

$s_{1,2} = \xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$\Rightarrow z_{1,2} = r\,\underline{/\pm\theta} = r\cos\theta \pm j r\sin\theta$

$z_{1,2} = 0.528 \pm j0.295$

$\Downarrow$

$r = \sqrt{0.528^2 + 0.295^2}$

$= 0.605$

$r = e^{-\xi\omega_n T} = e^{-T/\tau_c}$

$\ln r = -\dfrac{T}{\tau_c}$

$\ln(0.605) = \dfrac{-1}{\tau_c} \Rightarrow \tau_c \simeq \text{sec}$

$\tau_o = \frac{1}{2}(2) = 1$ sec.

* the desired poles of the observer.

$\tau_o = 1$ sec & $\xi = 1$

$\tau_o = \dfrac{1}{\xi \omega_n}$

$1 = \dfrac{1}{\omega_n} \Rightarrow \omega_n = 1$

$r = e^{-\xi \omega_n T} = e^{-\frac{T}{\tau_o}}$

$= e^{-1} = 0.3678$

$\theta = \omega_n T \sqrt{1-\xi^2} = 0$

$z_{1,2} = r\cos\theta \pm j \cancel{r \sin\theta}_{0}$

$= r = 0.3678$

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the desired ch. eq for the observer $\alpha_o(z)$:

$\alpha_o(z) = (z - 0.3678)^2$

$\alpha_o(z) = z^2 - 0.7356 z + 0.1353$

$\alpha_o(A) = A^2 - 0.7326 A + 0.1353 I$

$$= \begin{pmatrix} 0.3997 & 2.5288 \\ 0 & 0.3997 \end{pmatrix}$$

$G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $M_o \begin{pmatrix} C \\ CA \end{pmatrix}$

$M_o = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \Rightarrow M_o^{-1} = \begin{pmatrix} 1 & 0 \\ -0.5 & 0.5 \end{pmatrix}$

$G = \begin{pmatrix} 1.2644 \\ 0.1998 \end{pmatrix}$

Ex 2 : $x(k+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(k)$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

design a full order observer such that :

- the desired observer poles must be real
- the time constant is 10 sec
- $T = 1$ sec

$\xi = 1$ يستنتج

* desired poles of the observer:-

$$r = e^{-T/\tau} = e^{-1/10} = e^{-0.1} = 0.905$$

$$\theta = \omega_n T \sqrt{1-\xi^2} = 0 \quad (\text{Real and equal})$$

$$z_{1,2} = r = 0.905$$

$\xi = 1$

* the desired ch. eqn for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.905)^2 = z^2 - 1.81z + 0.819$$

$$\alpha_o(A) = \begin{pmatrix} 0.009 & 0.19 \\ 0 & 0.009 \end{pmatrix}$$

$$G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.19 \\ 0.009 \end{pmatrix}$$

$$M_o = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$M_o^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$